

Total Marks: 80

Hours: 3 hrs

Note : 1) Question no. 1 is compulsory.

2) Attempt any three questions out of five questions

Q-1

a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiples of each other. (05)

b) A function $f : R - \left\{ \frac{7}{3} \right\} \rightarrow R - \left\{ \frac{4}{3} \right\}$ is defined by $f(x) = \frac{4x-5}{3x-7}$, Prove that f is bijective

and find the rule for f^{-1} . (05)

c) Find $L \left[\frac{d}{dt} \left(\frac{1 - \cos 2t}{t} \right) \right]$ (05)

d) Prove that there does not exist an analytic function whose imaginary part is $3x^2 + \sin x + y^2 + 5y + 4$. (05)

Q-2

a) Find $L^{-1} \left[\frac{s}{(s^2+3^2)(s^2+5^2)} \right]$ using convolution Theorem. (06)

b) What is the chance of throwing ten with four dice? (06)

c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he gives answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. (08)

Q-3

a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? (06)

b) Find $L^{-1} \left(\tan^{-1} \left(\frac{2}{s^2} \right) \right)$ (06)

c) If R is the relation on the set of integers such that aRb if and only if $2a+3b$ is divisible by 5. Find the equivalence classes. (08)

Q-4

a) Evaluate $\int_{t=0}^{\infty} e^{-3t} \left(\frac{\cos(7t) - \cos(11t)}{t} \right) dt$ (06)

b) Find $L^{-1} \left[\frac{s^2 + 2s + 3}{(s^2 + 2s + 10)(s^2 + 2s + 17)} \right]$ (06)

c) Find the bilinear Transformation which maps the points $2, i, -2$ on to the points $1, i, -1$. Also find image of $|z| = 1$ of z-plane to w-plane. (08)

Q-5

a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? (06)

b) Find the analytic function $f(z) = u + iv$ in terms of z if $u - v = (x - y)(x^2 + 4xy + y^2)$. (06)

c) Solve $\frac{d^3 y}{dt^3} - 2\frac{d^2 y}{dt^2} + 5\frac{dy}{dt} = 0$ with $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$. (08)

Q-6

a) Prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ (06)

b) Draw the Hasse diagram of D_{105} . (06)

c) Find Laplace Transformation of the following

i) $te^{3t} \operatorname{erf}(\sqrt{t})$,

ii) $\sin t H(t) + (\cos t - \sin t) H(t - \pi)$ (08)