

(3 Hours)

[Total Marks: 80]

N.B (1) Question No. 1 is compulsory.

- (2) Solve any three questions out of remaining five questions.
- (3) Assumptions made should be clearly stated.
- (4) Figures to the right indicate full marks.

Q.1 (a) Two dice are rolled, find the probability that the sum is  
(i) Equal to 1 (ii) Equal to 4 (iii) Less than 13

[6M]

(b) Use the laws of logic to show that  
 $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$  is a tautology

[6M]

(c) Determine the matrix of the partial order of divisibility on the set A. Draw the Hasse diagram of the Poset. Indicate those which are chains

(1)  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

(2)  $A = \{3, 6, 12, 36, 72\}$

[8M]

Q.2 (a) Find the complement of each element in  $D_{42}$ .

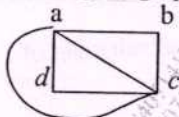
[6M]

(b) Let Q be the set of positive rational numbers which can be expressed in the form  $2^a 3^b$ , where a and b are integers. Prove that algebraic structure  $(Q, \cdot)$  is a group. Where  $\cdot$  is multiplication operation.

[6M]

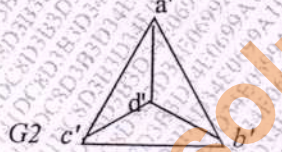
(c) Define isomorphic graphs. Show whether the following graphs are isomorphic or not.

[8M]



G1

Fig (a)

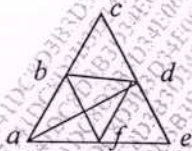


G2

Fig (b)

Q.3 (a) Determine which of the following graph contains an Eulerian or Hamiltonian circuit.

[6M]



Fig(a)



Fig(b)

(b) For all sets A, X and Y show that  
 $A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

[6M]

(c) Let  $f(x) = x+2$ ,  $g(x) = x-2$  and  $h(x) = 3x$  for  $x \in \mathbb{R}$ , Where  $\mathbb{R}$  = Set of real numbers. Find  
 $(g \circ f)$ ,  $(f \circ g)$ ,  $(f \circ f)$ ,  $(g \circ g)$ ,  $(f \circ h)$ ,  $(h \circ g)$ ,  $(h \circ f)$ ,  $(f \circ h \circ g)$

[8M]

Q.4 (a) Let R is a binary relation. Let  $S = \{(a, b) \mid (a, c) \in R \text{ and } (c, b) \in R \text{ for some } c\}$  Show that if R is an equivalence relation then S is also an equivalence relation.

[6M]

TURN OVER

(b) Determine the generating function of the numeric function  $a_r$ , where [6M]

- (i)  $a_r = 3^r + 4^{r+1}$ ,  $r \geq 0$   
 (ii)  $a_r = 5$ ,  $r \geq 0$

(c) Consider the (3, 6) encoding function  $e: B^3 \rightarrow B^6$  defined by [8M]

- $e(000) = 000000$   $e(001) = 001100$   $e(010) = 010011$   $e(011) = 011111$   
 $e(100) = 100101$   $e(101) = 101001$   $e(110) = 110110$   $e(111) = 111010$

Decode the following words relative to a maximum likelihood decoding function.  
 (i) 000101 (ii) 010101

Q.5 (a) Determine the number of positive integers  $n$  where  $1 \leq n \leq 100$  and  $n$  is not divisible by 2, 3 or 5. [6M]

(b) Use mathematical induction to show that  $1+5+9+\dots+(4n-3) = n(2n-1)$  [6M]

(c) Find the greatest lower bound and least upper bound of the set  $\{3, 9, 12\}$  and  $\{1, 2, 4, 5, 10\}$  if they exist in the poset  $(Z^+, /)$ . Where  $/$  is the relation of divisibility. [8M]

Q.6 (a) Let  $A = \{1, 2, 3, 4\}$  and Let  $R = \{(1,1) (1,2) (1,4) (2,4) (3,1) (3,2) (4,2) (4,3) (4,4)\}$ . Find transitive closure by Warshall's algorithm. [6M]

(b) Let  $H = \{[0]_6, [3]_6\}$  find the left and right cosets in group  $Z_6$ . Is  $H$  a normal subgroup of group of  $Z_6$ . [6M]

(c) Find the complete solution of the recurrence relation  $a_n + 2a_{n-1} = n+3$  for  $n \geq 1$  and with  $a_0 = 3$ . [8M]