

**Time: 3 Hours**

**Marks: 80**

- Note: 1) Q.1 is **COMPULSORY**.  
 2) Attempt **ANY 3 questions** from Q.2 to Q.6  
 3) Use of scientific calculators allowed.  
 4) Figures to right indicate marks.

**Q.1 a)** Find the Laplace transform of  $e^{-2t} t \cos t$  (05)

**b)** Find the inverse Laplace transform of  $\frac{3s+7}{s^2-2s-3}$  (05)

**c)** Determine whether the function  $f(z) = (x^3 + 3xy^2 - 3x) + i(3x^2y - y^3 + 3y)$  is analytic and if so find its derivative. (05)

**d)** Find the Fourier series for  $f(x) = x^2$  in the interval  $(-\pi, \pi)$ . (05)

**Q.2 a)** Evaluate  $\int_0^\infty \left( \frac{\sin 2t + \sin 3t}{t e^t} \right) dt = \frac{3\pi}{4}$  (06)

**b)** Find the Z- Transform of  $\left\{ \left( \frac{1}{4} \right)^{|k|} \right\}$  (06)

**c)** Show that the function  $v = e^x(x \sin y + y \cos y)$  is a harmonic function. Find its harmonic conjugate and corresponding analytic function. (08)

**Q.3 a)** From 8 observations the following results were obtained. (06)

$$\sum x = 59; \sum y = 40; \sum x^2 = 524; \sum y^2 = 256; \sum xy = 364.$$

Find the equation of the line of regression of x on y and the coefficient of correlation.

**b)** Find the bilinear transformation which maps the points  $z = -1, 0, 1$  onto the points  $w = -1, -i, 1$ . (06)

**c)** Obtain half-range sine series for  $f(x) = (x - 1)^2$  in  $0 < x < 1$ . Hence find  $\sum_{n=1}^\infty \frac{1}{n^2}$  (08)

**Q.4 a)** Find the inverse Laplace Transform by using convolution theorem  $\frac{1}{(s^2+a^2)(s^2+b^2)}$  (06)

**b)** Compute Spearman's Rank correlation coefficient for the following data: (06)

X	85	74	85	50	65	78	74	60	74	90
Y	78	91	78	58	60	72	80	55	68	70

c) Find the inverse Z-transform for the following; (08)

i)  $\frac{1}{(z-5)^2}$  ,  $|z| < 5$                       ii)  $\frac{z}{(z-2)(z-3)}$  ,  $|z| > 3$

**Q.5 a)** Using Laplace Transform evaluate  $\int_0^\infty e^{-t} (1 + 3t + t^2) H(t - 2) dt$  (06)

b) Prove that  $f_1(x) = 1$  ;  $f_2(x) = x$  ;  $f_3(x) = (\frac{3x^2 - 1}{2})$  are orthogonal over  $(-1, 1)$ . (06)

c) Solve using Laplace transform  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$  ,  $y = 2$  ,  $y' = 3$  at  $x = 0$ . (08)

**Q.6 a)** Find the complex form of Fourier series for  $f(x) = e^x$  ,  $(-\pi, \pi)$  . (06)

b) If  $u$  ,  $v$  are harmonic conjugate functions, show that  $uv$  is a harmonic function. (06)

c) Fit a straight line of the form  $y = a + bx$  to the following data and estimate the value of  $y$  for  $x = 3.5$ . (08)

x	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

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