

NB 1. Question No.I is compulsory.

2. Attempt any **three** from the remaining six questions.
3. Figures to the right indicate full marks.

Q1a. If $L\{t\sin\omega t\} = \frac{2\omega s}{(s^2+\omega^2)^2}$, find $L\{\omega t\cos\omega t + \sin\omega t\}$ [20]

b. If $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic, find a,b,c and d

c. Find the Fourier series expansion of $f(x) = x^3 \quad (-\pi, \pi)$

d. If the two regression equations are $4x - 5y + 33 = 0$, $20x - 9y - 107 = 0$ find i) the mean values of x and y

ii) the Correlation Coefficient iii) Standard deviation of y if variance of x is 9

Q2 a. Show that the function is Harmonic and find the Harmonic Conjugate

$$u = \cos x \cosh y - 2xy$$

[6]

b. Evaluate $\int_0^\infty e^{-t} (\int_0^t u^2 \sinh u \cosh u \, du) dt$ using Laplace Transform.

[6]

c. Find Fourier Series expansion of $f(x) = x \quad -1 < x < 0$
 $= x + 2 \quad 0 < x < 1$

[8]

Q3 a. Find the Analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$

[6]

b. Find Inverse Z transform of $\frac{5z}{(2z-1)(z-3)} \quad \frac{1}{2} < |z| < 3$

[6]

c. Solve the Differential Equation using Laplace transform

$$(D^2 - 2D + 1)y = e^t, y(0) = 2, y'(0) = -1$$

[8]

Q4 a. Find the Complex Form of Fourier Series for $f(x) = \cos ax \quad (-\pi, \pi)$

[6]

b. Find the Spearman's Rank correlation coefficient between X and Y.

[6]

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

c. Find the inverse Laplace transform of i) $\frac{s-1}{s^2+2s+2}$ ii) $\frac{e^{-\pi s}}{s^2(s^2+1)}$

[8]

Q5 a. Find the $Z\{f(k)\}$ where $f(k) = 4^k, k < 0$ [6]
 $= 3^k, k \geq 0$

b. Show that $\{\cos x, \cos 2x, \cos 3x, \dots\}$ is orthogonal set over the interval $[0, 2\pi]$.
 Construct the corresponding orthonormal set. [6]

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points
 $w = i, 0, -i$, Hence find the image of $|z| < 1$. [8]

Q6 a. Fit a straight line to the given data [6]

X	10	12	15	23	20
Y	14	17	23	25	21

b. Find Inverse Laplace Transform using Convolution theorem $\frac{1}{(s-2)^3(s+3)}$ [6]

c. Find Half Range Cosine Series for $f(x) = \sin x$ in $(0, \pi)$ and hence deduce that [8]
 $\frac{\pi^2 - 8}{16} = \frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots$
