

Total Marks: 80

Time Duration: 3Hr

- N.B.: 1) Question no.1 is compulsory.
 2) Attempt any three questions from Q.2 to Q.6.
 3) Figures to the right indicate full marks.

Maximum
 Marks

- Q1. a) Find the Laplace transform of $\frac{1}{t} e^{-t} \sin t$. [5]
 b) Find the inverse Laplace transform of $\frac{1}{\sqrt{2s+1}}$. [5]
 c) Show that the function $f(z) = \sinh z$ is analytic and find $f'(z)$ in terms of z . [5]
 d) Find the Fourier series for $f(x) = x$ in $(0, 2\pi)$. [5]

- Q2. a) Use Laplace transform to prove $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$. [6]
 b) If $\{f(k)\} = \begin{cases} 4^k, & k < 0 \\ 3^k, & k \geq 0 \end{cases}$, find $Z\{f(k)\}$. [6]
 c) Show that the function $u = \cos x \cosh y$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function. [8]

- Q3. a) Find the equation of the line of regression of Y on X for the following data. [6]

| | | | | | | | |
|---|----|----|----|----|----|----|----|
| X | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Y | 11 | 14 | 14 | 15 | 12 | 17 | 16 |

- b) Find the bilinear transformation which maps the points 1, -1, 2 on z-plane onto 0, 2, -i respectively of w-plane. [6]

- c) Find half range sine series for $f(x) = \begin{cases} x & , 0 < x < \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x < \pi \end{cases}$, Hence find the sum of

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)n^4}$$

- Q4. a) Find the inverse Laplace transform by using convolution theorem $\frac{1}{(s-a)(s+a)^2}$. [6]

- b) Calculate the coefficient of correlation between X and Y from the following data. [6]

| | | | | | | |
|---|---|---|----|----|----|---|
| X | 8 | 8 | 7 | 5 | 6 | 2 |
| Y | 3 | 4 | 10 | 13 | 22 | 8 |

- c) Find the inverse Z-transform of

i) $\frac{1}{(z-a)^2} \quad |z| < a$

ii) $\frac{1}{(z-3)(z-2)} \quad |z| > 3$

Q5.a) Using Laplace transform evaluate $\int_0^{\infty} e^{-t} (1 + 2t - t^2 + t^3) H(t-1) dt$. [6]

b) Show that set of functions $\cos x, \cos 2x, \cos 3x \dots$ is a set of orthogonal functions over $[-\pi, \pi]$. Hence construct a set of orthonormal functions. [6]

c) Solve using Laplace transform [8]
 $(D^3 - 2D^2 + 5D)y = 0$, with $y(0) = 0, y'(0) = 0, y''(0) = 1$.

Q6.a) Find the complex form of Fourier series for $f(x) = 2x$ in $(0, 2\pi)$. [6]

b) If $f(z)$ and $\overline{f(z)}$ are both analytic, prove that $f(z)$ is constant. [6]

c) Fit a curve of the form $y = ab^x$ to the following data. [8]

| | | | | | | |
|---|-----|-----|----|----|----|---|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| Y | 151 | 100 | 61 | 50 | 20 | 8 |

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